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Unique optimal solutions under circumstances of non-cooperative decentralization: Four Stackelberg game models applied to hotel-online travel agency channels

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Abstract

The paper develops four Stackelberg game models to explore the hotels-online travel agencies relationships to demonstrate unique optimal solutions under circumstances of non-cooperative decentralization. The specific Stackelberg game models are divided elementally into leaders and followers according to the roles played; and, they may be further sub-divided according to the types of decision variables involving commission rates and room rates. Results show that two of the four Stackelberg models used to determine the optimal commission rates and room rates exist. In the first model, the online travel agency, acting as the leader, determines the commission rate; and, the hotel, acting as the follower, determines the room rate. A unique optimal commission rate and room rate is therefore evident. In the second model, the hotel acting as the leader determines the commission rate and the online travel agency acting as the follower determines the room rate. Optimal commission rate and room rate exist as well. This business model has yet to be explored in future research.

Keywords: Stackelberg game, online travel agency, hotel, commission rate, non-cooperative decentralization

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1. Introduction

Many studies have used the Stackelberg game theory to explore the genuine interaction between the hotels and Online Travel Agencies (OTAs) (Chen, 2022; Guo et al., 2013; Liao et al., 2015; Liao et al., 2019; Ling et al., 2011; Ling et al., 2014). For example, Guo et al. (2013) explored the interaction between a single website and a limited number of hotels. In their model, the hotel acts as the leader that decides the commission to be paid; meanwhile, the website decides the effort level, which influences the eventual level of room sales for the hotels. In the Stackelberg game model used by Ling et al. (2014), the hotel provides the OTA with a contract, which proposes unit commission; afterwards, the OTA determines the ranking position of the hotel and the commensurate level of effort to be made if there is acceptance of the proposal. Although Guo et al. (2013) discussed the issue of commission and derived the optimal commission available, their study did not use the hotel room rate independently as a decision variable to solve the problem of optimality. Similarly, Ling et al. (2014) argued that the hotel determines the unit commission, and they took the room rate as an exogenous variable and did not consider it as a decision variable.

Although Liao et al. (2019) and Chen (2022) clarified the respective roles played within the Stackelberg game and the decision variables determined for each role, it is the OTA (considered as the leader) deciding the commission. Likewise, the hotel is assumed to be the follower which determines the room rate. However, previous research has not fully established a generalized framework for the respective roles played by both the hotels and OTAs (leaders or followers) along with the decisions (i.e., room rates or commission rates) made by hotels and OTAs. As such, the following cases are presented: Case 1) The OTA acts as the leader to determine the commission rate and the hotel acts as the follower to determine the room rate. Case 2) The hotel acts as the leader to determine the commission rate and the OTA acts as the follower to determine the room rate. Case 3) The OTA acts as the leader to determine the room rate and the hotel acts as the follower to determine the commission rate. Finally, Case 4) The hotel acts as the leader to determine the room rate and the OTA acts as the follower to determine the commission rate. To sum up, this paper has two definite research purposes. The first purpose is to outline a research framework that explores the binary roles acted out by the players (hotels / OTAs) along with the decisions (i.e., room rates or commission rates) made by hotels and OTAs within the Stackelberg game. The four models are obtained by means of permutation and combination. Secondly, an additional purpose is to verify whether these four Stackelberg models possess unique optimal solutions for their respective room rates and commission rates beneficial to hotel supply chain decision-makers.

2. Methodology

When considering the two sides of hotel supply chain (with one hotel and one OTA for purposes of this research), the cooperation model between hotel supply chain members is analogous that of Chen (2022). The Stackelberg game model discussed in this study is divided into the leaders and followers according to the roles played; so, if it is divided according to the types of decision variables, it can be further sub-divided into the commission rates and room rates to be used. This forms a 2x2 matrix. In this way, four different Stackelberg game models are obtained. r_i and p_i represent the commission rate and room rate respectively under the i-th case, denoted by (r_i, p_i). For example, (r_2, p_2) is used to represent Case 2 (the hotel acts as a leader to determine the commission rate whereas the OTA acts as a follower that determines the room rate). In the four Stackelberg game models provided, the hotel can act as either a leader or follower, and it can determine the room rate or the commission rate; similarly, the OTA can also act as either a leader or follower, and it can determine the room rate or the commission rate. Under the commission model, the hotel pays the OTA a commission fee $r_i p_i$ for every room sold by the OTA.

The demand for the hotel guest rooms is given as in Equation (1):

$$q(p_i)=\alpha-\beta p_i \quad (1)$$

where α is the scale parameter, α>0, and β is the price sensitivity coefficient of the demand function, β>0. For the sake of simplicity, α and β are assumed to be the same in all four models. The sales quantity $q(p_i)$ is non-negative and inversely proportional to the room rate p_i. The larger the sensitivity coefficient, the more sensitive the price change to the change in sales volume.

The profit function of the OTA is:

$$\pi_o=(r_i p_i - c_o)\ (\alpha - \beta p_i) \quad (2)$$

The profit function of the hotel is:

$$\pi_h=[(1- r_i)\ p_i - c_h]\ (\alpha - \beta p_i) \quad (3)$$

where π_o represents the profit of the OTA, and π_h the profit of the hotel; c_h and c_o represent the unit costs of the hotel and OTA respectively. The unit cost of the OTA is assumed to be less than that of the hotel. Next, proposition 1 to 4 are used to verify the existence of unique optimal room rates and commission rates for the four cases provided (Model 1 through Model 4).

Proposition 1. The OTA, acting as the leader, determines the commission rate r_1; the hotel, acting as the follower, determines the room rate p_1 (Model 1). There exist both a unique optimal commission rate and room rate (r_1^*, p_1^*) for Model 1.

Proof. By means of backward induction, the optimal decisions of the OTA and hotel according to the sequence of events are analyzed. The solution process is reversed. That is, the hotel's optimization problem is solved first. According to Equation (3),

$$\pi_h=[(1- r_1)p_1- c_h]\ (\alpha - \beta p_1)$$
$$=\alpha p_1-\beta p_1^2-\alpha p_1 r_1+\beta p_1^2 r_1-\alpha c_h+\beta c_h p_1$$

Differentiating the above formula with respect to p_1 as in Equation (4):

$$\frac{\partial \pi_h}{\partial p_1} = \alpha(1-r_1) - 2\beta(1-r_1)p_1 + \beta c_h \quad (4)$$

By setting Equation (4) to a value of zero, the optimal room rate p_1^* is obtainable as in Equation (5):

$$p_1^*(r_1) = \frac{\alpha}{2\beta} + \frac{c_h}{2(1-r_1)} \quad (5)$$

To show the optimality of the solution, it is demonstrable that the profit function of the hotel is concave in the room rate p_1. Differentiating Equation (4) with respect to p_1 again as in Equation (6):

$$\frac{\partial^2 \pi_h}{\partial {p_1}^2} = -2\beta(1-r_1) < 0 \text{, for } 0<r_1<1;\ \beta>0. \quad (6)$$

So, $\pi_h(p_1)$ is concave in p_1. Therefore, p_1^* is the unique optimal room rate for the hotelier.

Then, the existence of an optimal commission rate r_1 is to be proven. According to Equation (2),

$$\pi_o(r_1)= (r_1 p_1- c_o)\ (\alpha - \beta p_1) \quad (7)$$

Substituting Equation (5) into Equation (7), re-written as:

$$\pi_o(r_1) = [r_1(\frac{\alpha}{2\beta}+\frac{c_h}{2(1-r_1)})-c_o][\alpha-\beta(\frac{\alpha}{2\beta}+\frac{c_h}{2(1-r_1)})]$$

$$=\frac{r_1\alpha^2}{4\beta}-\frac{\alpha c_o}{2}+\frac{\beta c_h c_o}{2(1-r_1)}-\frac{r_1\beta c_h{}^2}{4(1-r_1)^2} \quad (8)$$

Differentiating Equation (8) with respect to r_1 as in Equation (9):

$$\frac{\partial\pi_o}{\partial r_1}=\frac{\alpha^2}{4\beta}+\frac{\beta c_h c_o}{2}\frac{1}{(1-r_1)^2}-\frac{\beta c_h{}^2}{4}[\frac{1}{(1-r_1)^2}+\frac{2r_1}{(1-r_1)^3}] \quad (9)$$

It is possible to demonstrate that the profit function of the OTA is concave in the commission rate r_1. Differentiating Equation (9) with respect to r_1 again as in Equation (10):

$$\frac{\partial^2\pi_o}{\partial r_1{}^2}=\beta c_h(c_o-c_h)\frac{1}{(1-r_1)^3}-\frac{3\beta c_h{}^2}{2}\frac{r_1}{(1-r_1)^4} \quad (10)$$

The unit cost of the OTA c_o is less than that of the hotel c_h according to the assumption previously, that is:

$$c_o < c_h$$

So, $[\beta c_h(c_o-c_h)\frac{1}{(1-r_1)^3}-\frac{3\beta c_h{}^2}{2}\frac{r_1}{(1-r_1)^4}]<0$, for $0<r_1<1$; $\beta>0$; $c_h>0$; $c_o>0$.

It can be proved that $\pi_o(r_1)$ is concave in r_1. Therefore, r_1^* is the unique optimal commission rate of the OTA. Consequently, a unique optimal commission rate and room rate (r_1^*, p_1^*) for Model 1 exists.

Proposition 2. The hotel, acting as the leader, determines the commission rate r_2; so, the OTA, acting as the follower, determines the room rate p_2 (Model 2). There exists a unique optimal commission rate and room rate (r_2^*, p_2^*) for Model 2.

Proof. The OTA's optimization problem is first solved by means of backward induction. According to Equation (2):

$$\pi_o=(r_2p_2-c_o)(\alpha-\beta p_2)$$
$$=\alpha r_2p_2-\beta r_2p_2^2-\alpha c_o+\beta c_op_2 \quad (11)$$

Differentiating Equation (11) with respect to p_2 as in Equation (12):

$$\frac{\partial\pi_o}{\partial p_2}=\alpha r_2-2\beta r_2p_2+\beta c_o \quad (12)$$

By setting the above formula to zero, the optimal room rate p_2^* can be obtained as in Equation (13):

$$p_2{}^*(r_2)=\frac{\alpha}{2\beta}+\frac{c_o}{2r_2} \quad (13)$$

It is possible to demonstrate that the profit function of the OTA is concave in p_2. Differentiating Equation (12) with respect to p_2 again as in Equation (14):

$$\frac{\partial^2 \pi_o}{\partial p_2^{\ 2}} = -2\beta r_2 < 0$$
, for $0<r_2<1$; $\beta>0$. (14)

So, $\pi_0(p_2)$ is concave in p_2. Therefore, p_2^* is the unique optimal room rate for the OTA.

Then, it is proved that there is a unique optimal solution for the commission rate. According to Equation (3),

$$\pi_h=[(1-r_2)p_2-c_h](\alpha-\beta p_2)$$
$$=(\alpha-\alpha r_2+\beta c_h)p_2+(\beta r-\beta)p_2^2-\alpha c_h \quad (15)$$

Substituting Equation (13) into Equation (15):

$$\pi_h=(\alpha-\alpha r_2+\beta c_h)(\frac{\alpha}{2\beta}+\frac{c_o}{2r_2})+(\beta r_2-\beta)(\frac{\alpha}{2\beta}+\frac{c_o}{2r_2})^2-\alpha c_h$$
$$=(\frac{\alpha^2}{4\beta}-\frac{\alpha^2}{2\beta})r_2+(\frac{\beta c_h c_o}{2}+\frac{\beta c_o^{\ 2}}{4})\frac{1}{r_2}-\frac{\beta c_o^{\ 2}}{4}\frac{1}{r_2^{\ 2}}+\frac{\alpha^2}{2\beta}+\frac{\alpha c_h}{2}-\frac{\alpha^2}{4\beta}-\alpha c_h \quad (16)$$

Differentiating Equation (16) with respect to r_2 as in Equation (17):

$$\frac{\partial \pi_h}{\partial r_2}=(\frac{\alpha^2}{4\beta}-\frac{\alpha^2}{2\beta})-(\frac{\beta c_h c_o}{2}+\frac{\beta c_o^{\ 2}}{4})\frac{1}{r_2^{\ 2}}+\frac{\beta c_o^{\ 2}}{2}\frac{1}{r_2^{\ 3}} \quad (17)$$

Differentiating Equation (17) with respect to r_2 again to demonstrate the optimality as in Equation (18):

$$\frac{\partial^2 \pi_h}{\partial r_2^{\ 2}}=(\beta c_h c_o+\frac{\beta c_o^{\ 2}}{2})\frac{1}{r_2^{\ 3}}-\frac{3\beta c_o^{\ 2}}{2}\frac{1}{r_2^{\ 4}}$$
$$=\frac{\beta c_o^{\ 2}}{r_2^{\ 3}}(\frac{c_h}{c_o}+0.5-\frac{1.5}{r_2}) \quad (18)$$

$1.5/r_2$ is divided into $(1/r_2+0.5/r_2)$. Substituting into Equation (18), re-written as in Equation (19):

$$\frac{\partial^2 \pi_h}{\partial r_2^{\ 2}}=\frac{\beta c_o^{\ 2}}{r_2^{\ 3}}(\frac{c_h}{c_o}+0.5-\frac{1}{r_2}-\frac{0.5}{r_2})$$
$$=\frac{\beta c_o^{\ 2}}{r_2^{\ 3}}[(\frac{c_h}{c_o}-\frac{1}{r_2})+(0.5-\frac{0.5}{r_2})] \quad (19)$$

According to the assumption, the unit cost of the hotel c_h is less than that of the entire channel c , that is, $c_h < c$, re-written as:

$$\frac{c_o}{c_h}>\frac{c_o}{c} \quad (20)$$

According to Chen (2022), the optimum commission rate exists as in Equation (21):

$$r_2 = \frac{c_o}{c} \tag{21}$$

Substituting Equation (21) into Equation (20) as follows:

$$\frac{c_o}{c_h} > r_2 \tag{22}$$

Reciprocal of the above formula is re-expressed as in Equation (23):

$$\frac{c_h}{c_o} < \frac{1}{r_2} \tag{23}$$

And, $[0.5-(0.5/r_2)]<0$, for $0<r_2<1$.

Substituting the above results and Equation (23) back into Equation (19) as in Equation (24):

$$\frac{\partial^2 \pi_h}{\partial {r_2}^2} < 0 \tag{24}$$

So, $\pi_h(r_2)$ is concave in r_2, Therefore, r_2^* is the unique optimal commission rate for the hotel. Consequently, there exist unique optimal solutions (r_2^*, p_2^*) for Model 2.

Proposition 3. The OTA, acting as the leader, determines the room rate p_3; so, the hotel, acting as the follower, determines the commission rate r_3 (Model 3). A unique optimal commission rate and room rate (r_3^*, p_3^*) do not exist for Model 3.

Proof. The hotel's optimization problem is first solved by means of backward induction.
According to Equation (3),

$$\begin{aligned}\pi_h &= [(1-r_3)p_3-c_h](\alpha-\beta p_3)\\ &= \alpha p_3-\beta p_3^2-\alpha p_3 r_3+\beta p_3^2 r_3-\alpha c_h+\beta c_h p_3\end{aligned} \tag{25}$$

Differentiating Equation (25) with respect to r_3 as in Equation (26):

$$\frac{\partial \pi_h}{\partial r_3} = \beta {p_3}^2 - \alpha p_3 \tag{26}$$

The first-order partial differential is not found to be a function of the commission rate r_3, so the optimum commission rate r_3^* cannot be found. Then, the second order partial differential is derived as in Equation (27):

$$\frac{\partial^2 \pi_h}{\partial {r_3}^2} = 0 \tag{27}$$

The results show that r_3 is not a unique optimal solution. Consequently, unique optimal solutions (r_3^*, p_3^*) do not exist for Model 3.

Proposition 4. The hotel, acting as the leader, determines the room rate p_4; so, the OTA, acting as the follower, determines the commission rate r_4 (Model 4). A unique optimal commission rate and room rate (p_4^*, r_4^*) does not exist for Model 4.

Proof. The OTA's optimization problem is first solved by means of backward induction. According to Equation (2),

$$\pi_o=(r_4p_4-c_o)(\alpha-\beta p_4)$$
$$=\alpha r_4p_4-\beta r_4p_4^2-\alpha c_o+\beta c_op_4 \quad (28)$$

Differentiating Equation (28) with respect to r_4 as in Equation (29):

$$\frac{\partial \pi_o}{\partial r_4}=\alpha p_4-\beta {p_4}^2 \quad (29)$$

The first-order partial differential is not found to be a function of the commission rate r_4, as such the optimum commission rate r_4^* cannot be found. Then, the second order partial differential is derived as in Equation (30):

$$\frac{\partial^2 \pi_o}{\partial {r_4}^2}=0 \quad (30)$$

The result of Equation (30) shows that r_4 is not a unique optimal solution. Consequently, unique optimal solutions (r_4^*, p_4^*) do not exist for Model 4.

Of the four Stackelberg models above, Model 1 and Model 2 exhibit optimal commission rates and hotel room rates (r_1^*, p_1^*) and (r_2^*, p_2^*), while Models 3 and 4 do not provide optimal commission rates and hotel room rates.

3. Conclusion

3.1. Theoretical Contribution

This research has several theoretical contributions. First, it establishes a framework for the generalization of the hotel supply chain wherein the binary roles are acted out by the players (hotels/OTAs) in the Stackelberg game. Likewise, in this game context, two essential decisions (room rates and commission rates) are made by these two roles. Accordingly, four Stackelberg game models were discussed in depth, and results were obtained regarding the optimal commission rates and room rates. Two of the four Stackelberg models examined prove that optimal commission rates and room rates exist in some form. Second, in Model 1 the OTA, acting as the leader, determines the commission rate; so, the hotel, acting as the follower, determines the room rate. In Model 2, the hotel, acting as the leader, determines the commission rate; so, the OTA, acting as the follower, determines the room rate. In both cases, unique optimal commission rates and room rates exist. According to the results of this study, the researchers can follow Model 1 and Model 2 to find the analytical solution of optimal commission rates and guest room rates. Third, Model 2 established in this study is a new model that has not previously been proposed in the literature, so a new research topic can be developed based on such a new cooperation model.

3.2. Managerial Implications and Limitations

OTAs have played an important role in the hotel industry for years. This study helps hotel owners/managers and the OTAs develop a greater number of cooperative relationships. Coordination of the hotel supply chain makes the cooperation between the members smoother, which helps the entire hotel supply chain to achieve higher profitability. This article assists hotel owners to handle the frequently contentious issue of over-paying OTA commission fees by determining the optimum commission rate. The new cooperation model constructed as part of this study may support better performance than the previous proposed cooperation model used among the hotel supply chain members to optimize channel-wide profitability and/or channel efficiency.

However, the theoretical nature of this study has its limitations. First, this study is conceptual and no empirical data is provided. Second, the demand for hotel rooms is assumed to be a deterministic, price-sensitive linear function. Future studies may consider more general demand functions. According to the framework constructed in this paper, the possible future analytical solutions of Model 1 and Model 2 are derivable and solvable for obtaining the optimal commission rates and room rates under the non-cooperative decentralization scheme. Practical issues and examples of the analytical solutions according to Model 1 and Model 2 can be verified in future studies.

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